حول شبه الدوال المستمرة البسيطة

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الملخص العربي :

يقوم قسم توبولوجيا الرياضيات بدراسة الخصائص التي لا تتغير بشكل متزايد في خطوات ذات أرقام صحيحة ، بدلاً من تغييرها باستمرار ، وللتوبولوجيا دور مهم جدا في انعاش العلوم الأخرى مثل علم الفيزياء ، حيث حصل ثلاثة علماء على جائزة نوبل لأبحاثهم في فيزياء المواد المكثفة ، لعملهم على التوبولوجيا العابرة (التحولات الطورية التوبولوجية والأطوار التوبولوجية للمادة) , وهي ظاهرة تقف وراء الحالات الغريبة للمادة مثل الموصلات الفائقة والموائع الفائقة والطبقات المغناطيسية الرقيقة ، وقد كشف عملهم عن آفاق جديدة تتعلق بسلوك المواد التي تسمى العوازل التوبولوجية ، والتي قد تجعل من الممكن تصنيع أجهزة الكمبيوتر مع كمية أكثر تعقيدًا وتعقيدًا. لقد أظهرت أبحاث العلماء أهمية كبيرة في تطوير وفهم العوازل الطوبوغرافية ، حيث أثبتت الأبحاث التي أجراها هؤلاء العلماء أنها ذات أهمية كبيرة في تطوير وفهم العوازل التوبولوجية ، والمواد الجديدة التي تمنع تدفق الإلكترونات في أجزائها الداخلية بينما تسمح بنقل الكهرباء عبر أسطحها . هذه الخاصية الفريدة يمكن أن تجعل العوازل التوبولوجية مفيدة جدا في الوصول إلى أنواع جديدة من الجسيمات الأولية وفي تكوين الدوائر داخل أجهزة الكومبيوتر.

وفي هذه الورقة نتناول المجموعة شبه المفتوحة والمجموعة شبه المغلقة في الفضاء التوبولوجي حيث تستخدم لتعريف الدالة شبه المستمرة، والدالة شبه البسيطة المستمرة ، والدالة البسيطة شبه مستمرة. نحن أيضا نقدم مفاهيم مجموعات شبه المتراصة وشبه المصفاة ، وعلاوة على ذلك يتم التحقيق في بعض خصائص هذه الدوال .

On Semi Simply Continuous Functions

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Abstract

Topology branch of mathematics studies the properties that do not change only increasingly in steps with correct numbers, rather than changing continuously. Topology has a very significant role in the recovery of other sciences such as physics, where three scientists received the Nobel Prize for their research in the physics of condensed materials, for their work on transient topological. Their work has revealed new horizons concerning with the behavior of the materials called topological insulators, which may make it possible to manufacture computers with more complex and sophisticated quantity. These scientists' researches have shown great importance in the development and understanding of topological insulators.

The research of these scientists has proved to be of great importance in the development and understanding of topological insulators, new materials that prevent the flow of electrons in their internal parts while allowing the transmission of electricity across their surfaces. This unique property can make topological insulators very useful in arriving at new types of elementary particles and in the composition of the circuitry within the quantum computers.

In this paper we defined simply semi-open, simply semi continuous function, and semi-simply continuous function. We also introduced the concepts of semi-compact sets and semi-filter. Furthermore, some properties of these functions are investigated.

Objectives

1 - Study groups close to opening and closing because they are in fact scientific

more accurate.

2. The development of new types of such groups.

3. Create new relationships based on these groups.

Conclusions and recommendations

In this paper, we succeeded in obtaining a new family of semi-open groups that simply adopts the concepts of open open set, as well as studying a new concept, semi-filter and semi-compactness. We presented examples of its existence, studied its properties and its relation to the corresponding concepts, results in establishing new types of communication and studying their characteristics. The future work after this topic is to use the results of this research and previous research to prepare a new research in which we study a new relationship of the topological relations with the theory of the group is β-Rough Sets

Kay Words

Simply semi-open, simply semi continuous function, semi-simply continuous function, semi-compact sets and semi- filter.

1- Introduction

Levine N.[6] introduced the concept of semi-continuous function in 1963. Some of definitions and theorems with respect to a base for topology, neighborhood, compact space, closure operator and quasi-discrete topology have been handled in [2,5,7,10,11]. In this paper we introduce new type of classes named simply semi-open as a generalization of simply open sets, new types of continuous functions between topological spaces, and we used the new types of sets to introducing some types of semi-compact sets and semi-filters in topological spaces.

Definition 1.1 [**13**]

A filter $\mathcal{F}$ on a set S is a non-empty collection of non-empty subset of $S$with the properties

1- If $F_1 , F_2 \in \mathcal{F}$ then $F_1 \cap F_2 \in \mathcal{F}$.

2- If $B_1, B_2 \in T$ then $B_1 \cap B_2 \in T, \forall B_1, B_2$.

3- If $F \in \mathcal{F}$ &$F \subset G$, then $G \in \mathcal{F}$, $G \subseteq S$.

Definition1.2

A semi filter $\mathcal{F}_S$ on a set X is a non-empty collection of non-empty subset of X with the properties

1- If $f_{s_1} , f_{s_2} \in \mathcal{F}_S$then $f_{s_1} \cap f_{s_2} \in \mathcal{F}_S$.

2- If $f_s \in \mathcal{F}_S$& $f_s \subset G$, then $G \in \mathcal{F}_S, G \subseteq X$.

Properties of closure of any set [**5**]

If (X, τ) is a topological space, $A , B \subseteq X$, then

1- $A \subset \overline{A}$ s.t. $\overline{A} = \bigcap\{F: F\ closed , A \subset F\}$.

2- $\overline{\Phi} = \Phi$, $\overline{X} = X$.

3- $\overline{A \cup B} = \overline{A} \cup \overline{B}$.

4- $\overline{A \cap B} \subset \overline{A} \cap \overline{A}$.

5- $\overline{\overline{A}} = \overline{A}$.

Properties of interior of any set [**5**]

If (X, τ) is topological space, $A , B \subseteq X$, then

1- $A^{\circ} \subset A\ s.t.\ A^{\circ} = \bigcup\{G \subset X, G \in \tau, G \subset A\}$.

2- $X^{\circ} = X$, $\emptyset^{\circ} = \emptyset$.

3- $(A \cap B)^{\circ} = A^{\circ} \cap B^{\circ}$.

4- $(A \cup B)^{\circ} \supset A^{\circ} \cup B^{\circ}$.

5- $(A)^{\circ^{\circ}} = A^{\circ}$.

2- Semi continuous functions in topological spaces

In this section we introduce the concept of simply $S-open$ (for short, $S^MS-open$) set in topological spaces by using of simply open sets (for short S^M-open). We use these concept to introduce type of semi-continuous function in topological space.

Definition 2.1[**6**]

A sub set A of topological space (X,τ) is called semi open set if$A \subseteq cl\big(int(A)\big)$.

Definition 2.2 [**9**]

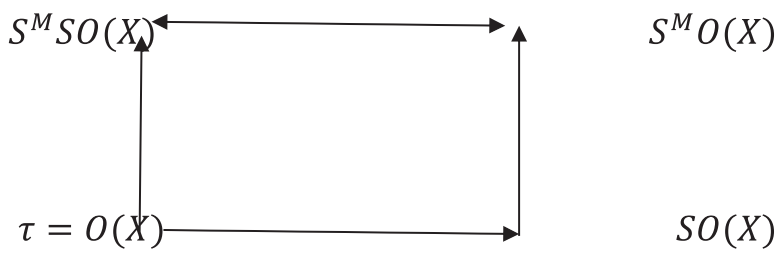
A sub set A of topological space (X,τ) is called simply open set (resp. S^M-open) if $A = G \cup N$ where G is open set and N is nowhere dense, where $cl\big(int(A)\big) = \emptyset$.

Definition 2.3

A sub set A of a topological space (X,τ) is called simply semi open (resp. $S^MS-open$) if $A = G \cup N$, where G is semi open set and N is nowhere dense.

Remark 2.1

The relation between $S^MSO(X)$ and some types of near open sets shown in the following diagram.



Example 2.1

Let $X = \{a, b, c, d\}, \tau = \{X, \emptyset, \{d\}, \{a, d\}\}$, then

$SO(X) =$
$\{X, \emptyset, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, c, d\}, \{a, b, d\}, \{b, c, d\}\}$,

$S^M O(X) = \begin{Bmatrix} X, \emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \\ \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\} \end{Bmatrix}$,

$S^M SO(X) = \begin{Bmatrix} X, \emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \\ \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\} \end{Bmatrix}$,

&

$\{c, d\} \in SO(X)\ but\ \{c, d\} \notin \tau$, $\{b, c\} \in S^M SO(X)\ but\ \{b, c\} \notin \tau$, $\{a, b\} \in S^M SO(X)\ but\ \{a, b\} \notin SO(X)$.

Note that in this example $S^M O(X) = S^M SO(X)$.

Theorem 2.1 [**1**]

If A is an open subset of a space X, then $Cl_S(A) = int(cl(A))$, where $Cl_S(A)$

Semi-closure of A.

Definition 2.4 [4]

A space X is semi-totally continuous if the inverse image of every semi-open subset of Y is clopen in X.

Definition 2.5

A function $f: (X, \tau) \longrightarrow (Y, \mu)$ is called

1- Semi-continuous ($S - continuous$) iff $f^{-1}(G) \in SO(X)$for every $G \in \mu$.

2- Simply S-Continuous ($S^M S -$ Continuous) iff$f^{-1}(G) \in S^M SO(X)$ for every

$G \in \mu$.

3- Semi-Simply continuous ($S.S^M - continuos$) if $f^{-1}(G) \in S^M SO(X)$ for every $G \in SO(Y)$.

Example 2.2

Let $X = \{a, b, c, d, e\}$, $Y = \{u, v, z, w, k\}$,
$\tau = \{X, \emptyset, \{a\}, \{a, b\}, \{a, c, d\}, \{a, b, c, d\}\}$&
$\mu = \{Y, \emptyset, \{u\}, \{u, v\}, \{u, v, z\}, \{u, z, w\}, \{u, v, k\}\}$.
Let $f: (X, \tau) \longrightarrow (Y, \mu)$ defined by $f(a) = u$, $f(b) = v$, $f(c) = w$, $f(d) = k$,
$f(e) = z$ &

$$SO(X) = \left\{\begin{matrix} X, \emptyset, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, e\}, \{a, b, c\}, \\ \{a, b, d\}, \{a, b, e\}, \{a, c, d\}, , \{a, c, e\}, \{a, d, e\}, \\ \{a, b, c, d\}, \{a, b, c, e\}, \{a, b, d, e\}, \{a, c, d, e\} \end{matrix}\right\}.$$

We have f is $s-continuous$ but not continuous, since there exist$\{u, v, z, w\} \in \mu$ such that $f^{-1}(\{u, v, z, w\}) = \{a, b, c, e\} \notin \tau$.

Example 2.3

Let $X = \{a, b, c, d\}$, $Y = \{u, v, z, w\}$, $\tau = \{X, \emptyset, \{a\}, \{b, c\}\}$, and
$\mu = \{Y, \emptyset, \{u\}, \{v\}, \{u, v\}, \{v, z, w\}\}$. Let $f: (X, \tau) \longrightarrow (Y, \mu)$ defined by $f(a) = u$, $f(b) = w$, $f(c) = z$ and $f(d) = v$ &
$SO(X) = \{X, \emptyset, \{a\}, \{a, d\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}\}$,

$$S^M SO(X) = \left\{\begin{matrix} X, \emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, d\}, \{b, c\}, \\ \{b, d\}, \{c, d\}, \{a, b, c\}, \{b, c, d\} \end{matrix}\right\}$$

, we have f is $S^M S-$ Continuous but not $S-continuous$, since there exists $\{v\} \in \mu$ such that $f^{-1}(\{v\}) = \{d\} \notin So(X)$. We have $SO(Y) = \{Y, \emptyset, \{u\}, \{v\}, \{u, v\}, \{v, z\}, \{v, w\}, \{u, v, z\}, \{u, v, w\}, \{v, z, w\}\}$, and it is f is $S^M S - continuos$ but not $S.S^M - continuos$, since there exist $\{u, v\}$ &
$\{u, v, z\} \in SO(Y) \ni f^{-1}(\{u, v\}) = \{a, b\}$& $f^{-1}(\{u, v, z\}) = \{a, c, d\} \notin S^M SO(X)$.

3- Semi-filter and semi-Compactness

In This section we shall state (special) types of compact a spaces, semi-filter and their properties.

Definition 3.1 [3]

Let $K = (U, R)$ be a general knowledge base,R is any binary relation on U, then we can define two new approximations namely, semi-lower and semi-upper approximation as follows

$$\underline{S}(X) = X \cap \overline{R}\left(\underline{R}(X)\right) \quad , \quad \overline{S}(X) = X \cup \underline{R}\left(\overline{R}(X)\right).$$

Definition 3.2 [8]

Let $\mathcal{A} = \{G_i\}$be a class of subset of X such that $A \subset \bigcup_i G_i \; for \; some \; A \subset X$. Really that $\mathcal{A}$ is then called a cover of A, and an open cover if each G_i is open. Furthermore, if a finite subclass of $\mathcal{A}$ is also a cover of A i.e. if $\exists\, G_{i1}, G_{i2}, .., G_{im} \in \mathcal{A}$ such that $A \subset G_{i1} \cup G_{i2} \cup \ldots \cup G_{im}$ then $\mathcal{A}$ is said to be reducible to a finite cover, or contains a finite sub cover.

Definition 3.3 [8]

A subset A of a topological space X is compact if every open cover of A is reducible to a finite cover. In other words, if A is compact and $A \subset \bigcup_i G_i$, where the G_i are open sets, then one can select a finite number of the open sets, say

$G_{i1}, G_{i2}, \ldots , G_{im}$, so that $A \subset G_{i1} \cup G_{i2} \cup \ldots \cup G_{im}$.

Definition 3.4 [12]

A sub set A of a topological space (X, τ) is called regular open set if

$A = int\left(cl(A)\right)$.

Definition 3.5 [2]

A topological space (X, τ) is called compact space if every open cover of X has a finite sub cover.

Definition 3.6

A topological space $(X,\tau)$is called semi-compact $(S-compact)$ if every semi-open cover of X has a finite sub cover.

Remark 3.1

Every S-compact is compact.

Example 3.1

A discrete space is $S-compact \Leftrightarrow$ it is finite. That is an infinite discrete space is not $S-compact$, since $\{\{x\}: x \in X\}$ is semi-open cover with no finite sub cover.

Theorem 3.1

The following are equivalent

i- X is a $S-compact$.

ii- Every semi-filter has a cluster point.

Proof

i→ ii Since X is a $S-compact$ then there exist $S-open$ cover has a finite sub cover. Let $\mathcal{F}$ be any filter on X and $\mathcal{A}$ be aclosed collectionsuch that $\bigcap_{C\in\mathcal{A}} C = \emptyset$ then we get $\bigcup_{C\in\mathcal{A}} C^C = X$ so $\{C^C\}_{C\in\mathcal{A}}$ is an open cover to the space X but $\{C^C\}_{C\in\mathcal{A}}$ has no finite sub cover since $c_1, c_2, \dots, c_n \in \mathcal{A}$ with $X = {c_1}^C \cup {c_2}^C \cup \dots \cup {c_n}^C$ then $\bigcap_{i=1}^{n} C_i = \emptyset$.A contradiction. Then every closed collection has a non-empty intersection where $\overline{C_i} \in \mathcal{F}$ for all$i = 1,2,..,n$ and so $\bigcap_{i=1}^{n} \overline{C} \in \mathcal{F}$, there for $\{\overline{C}: C \in \mathcal{F}\}$ has a finite intersection property so every semi filter has a cluster point.

ii → i since every semi filter has a cluster point then $\{\overline{F}: F \in \mathcal{F}\}$ is a closed collection and if $f_1, f_2, \dots, f_n \in \mathcal{F}$ then $\overline{F_i} \in \mathcal{F} \forall i = 1,2, \dots,$ and so$\bigcap_{i=1}^{n} \overline{F_i} \neq \emptyset$, suppose that X is not $S-compact$

then there exist an open cover $\mathcal{A}$ which has no finite sub cover that is for $\mathcal{A}$,we have $X = \bigcup_{i=1}^{n} A_i$ which is equivalent to$\bigcap_{i=1}^{n} A_i^C \neq \emptyset$ for any finite sub collection$\{A_1, A_2, \dots, A_n\}$ from $\mathcal{A}$.

Definition 3.7 **[13]**

1- A space X is $H - closed$ iff every open filter has a cluster point.

2- A space X is $H - closed$ iff every open cover has a finite sub collection whose closures cover (i.e. a finite dense sub system). If an open filter dose not have a cluster point, the complement of closures of its elements from an open cover with no finite dense sub system.

3- An $H - closed$ space is compact iff it is regular.

Definition 3.8

1- A space X is $H - semiclosed$ iff every open semi-filter has a cluster point.

2- An $H - semi - closed$ space is $S - compact$ iff it is regular.

Definition 3.9

A space $(X, \tau)$is said to be simply compact $(S^M - compact)$ if every simply open cover of X has a finite sub cover.

Example 3.2

Let $X = \{a, b, c, d\}$, $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c, d\}\}$,

$\overline{\tau} = \{X, \emptyset, \{a\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$,

$SO(X) =$

$\{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}\}$,

$$S^M O(X) = \begin{Bmatrix} X, \emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \\ \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\} \end{Bmatrix}$$

Then we have X is semi-compact.

***Example 3.3* [13]**

1- $\mathcal{R}$ is not compact. In fact, the cover of $\mathcal{R}$ by the open sets $(-n, n)$, for $n \in \mathcal{N}$, can have not finite sub cover.

***Theorem 3.2* [13]**

The continuous image of a compact space is compact.

Theorem 3.3

The continuous image of a $S-compact$ space is $S-compact$.

Proof

Suppose that (X,τ) be a $S-compact$ space and $f:(X,\tau) \rightarrow (Y,\mu)$ is a continuous function. Let $\{V_i: i \in I\}$ be a semi open cover of $f(A)$ such that
$f(A) \subset Y, A \subset X \,\&A \subset f^{-1}\big(f(A)\big) \subset f^{-1}(\cup\{V_i: i \in I\}) \subset \cup(\{f^{-1}(V_i): i \in I\})$.
Since f is continuous, then $\{f^{-1}(V_i): i \in I\} \subset SO(X)$, since X is $S-compact$ and $\{f^{-1}(V_i): i \in I\}$ is an $S-open$ cover of X, then there exist a finite sub set I_o of I such that $A \subset \cup\{f^{-1}(V_i): i \in I_o\}$, $\Rightarrow f(A) \subset \cup\{ff^{-1}(V_i): i \in I_o\} \subset \cup\{V_i: i \in I_o\}$.
Thus $f(Y,\mu)$ is $S-compact$.

***Theorem 3.4* [13]**

Every closed sub set of a compact is compact.

Theorem 3.5

A $Semi-closed$ sub set of a $S-compact$ space (X,τ) is $S-compact$.

Proof

Let $A \in SC(X)$, then $X - A \in SO(X)$. Let $\{G_i: i \in I\} \subset SO(X)$ be a semi-open cover of A, since X is $S-compact$ then there exist a finite sub cover

$\mathcal{B} = \{G_i : i \in I_o\}$ such that $X = (X - A) \cup \left(\bigcup_{i \in I_o} G_i\right) \in SO(X)$. Hence $A \subset \bigcup_{i \in I_o} G_i$ and A is $S - compact$ sub set of X.

Theorem 3.6

Let $A \,\&\, B$ be sub set of a topological space (X, τ). If A is $S - compact$ with
respect to X and B is $S - closed$ set in(X, τ), then $A \cap B$ is $S - compact$ with respect to X.

Proof

Suppose that $\{U_i : i \in I\}$ be an semi-open cover of $A \cap B$, and since $B \in SC(X)$
then $(X - B) \in SO(X)$, $(X - B) \cup \{U_i : i \in I\} \subset SO(X)$ which is cover of A. And since A is $S - compact$ w.r.t. X, then there exist a finite sub cover $\{U_i : i \in I_0\}$
such that $A \subset \{U_i : i \in I_0\} \cup (X - B)$.
Then $A \cap B \subset \{U_i : i \in I_0\} \cup (X - B) \cap B \subset \bigcup\{U_i : i \in I_0\}$.Thus $A \cap B$ is
$S - compact$ with respect to X.

Theorem 3.7

Every $S - open$ subset of X is $S - compact$ iff it is $S - compact$ with respect to X.

Proof

Let A be $S - compact$, and let $\{U_i : i \in I\}$ be an $S - open$ cover of A. Since A is $S - compact$ then $A \subset \bigcup\{U_i : i \in I\}$, and hence there exact a finite subset sub cover $\{V_i : i \in I\}$ such that $A \subset \bigcup\{V_i : i \in I\}$. Hence A is $S - compact$ relative to X. Conversely, let $\{W_i : i \in I\}$ be $S - open$ cover of A, since A is $S - compact$ set relative to$X$. Thus $A \subset \bigcup\{W_i : i \in I\}$ and hence there exist a finite sub cover
$\{B_i : i \in I\}$ such that $A \subset \bigcup\{B_i : i \in I\}$. Then A is $S - compact$.

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